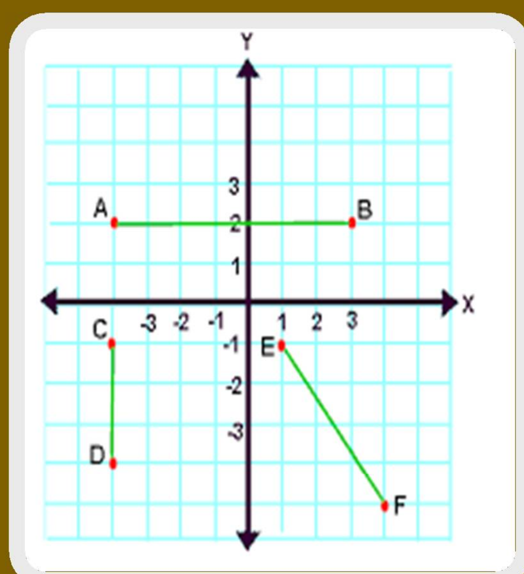
SSLC CLASS NOTES:
CHAPTER-14

COORDINATE GEOMETRY


René Descartes
(1596-1650)



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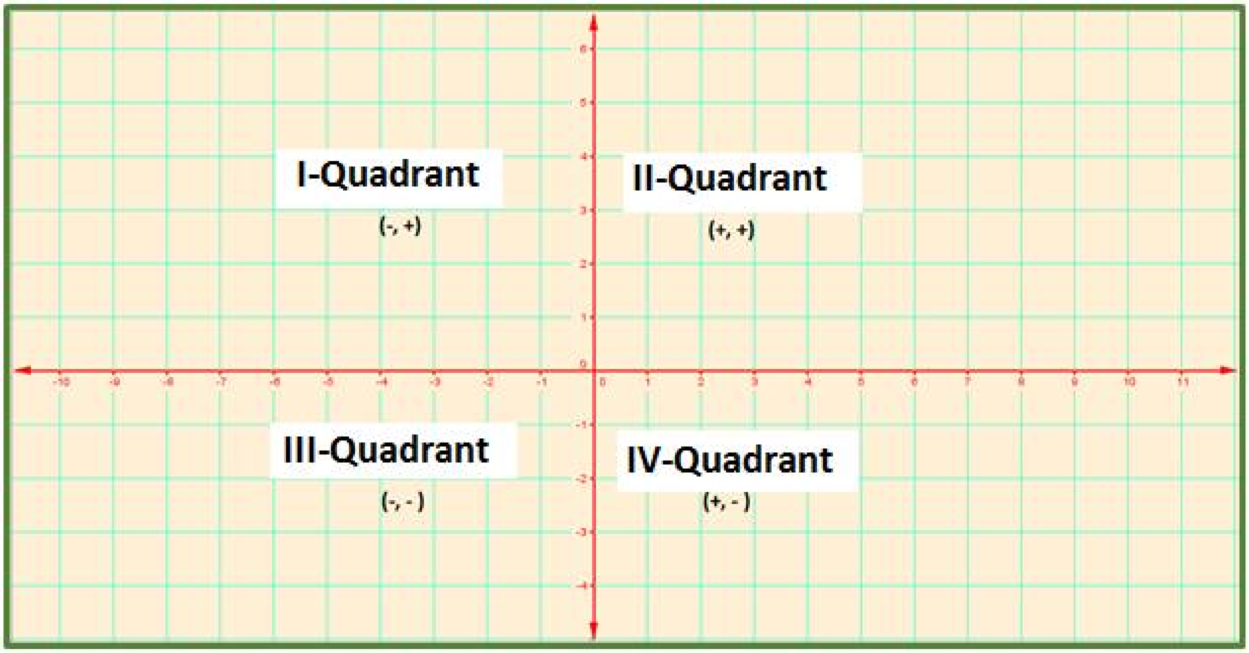
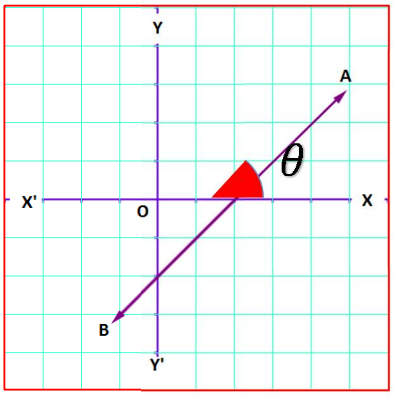


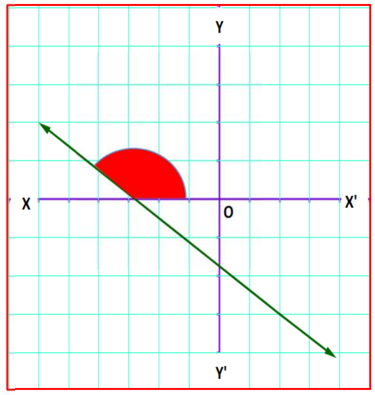
Coordinate Axes	The two lines perpendicular to each other
X'OX	X – Axis (Horizontal line)
Y'OY	Y – Axis (perpendicular line)
Origin: (O)	The point of intersection of two axes
Quadrants	Coordinate axes devides the plane into four parts. Each part is called quadrants
X- Coordinate(ABSCISSA)	The perpendicular distance from y-axis
y-Coordinate (Ordinate)	The perpendicular distance from x-axis
(0,0)	Coordinates of the origin
(x,0)	Coordinates of any point on X-axis
(0,y)	Coordinate of any point on Y-axis

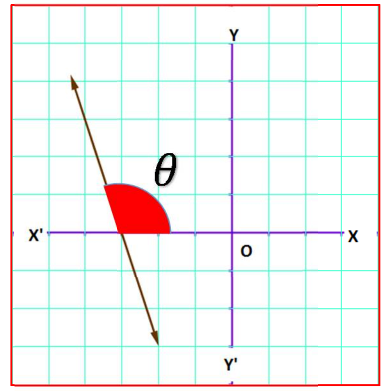
Inclination of a straight line:

The angle formed at the point of intersection with the X-axis which is in positive direction from the X-axis to the line

The angle is measured in anti-clockwise direction is called positive direction







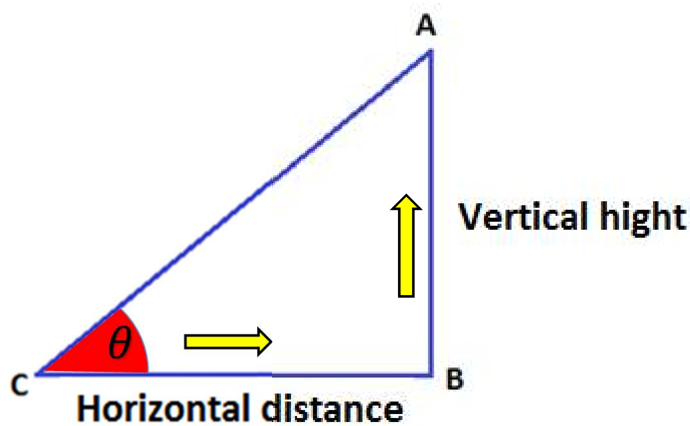
Slope:

The ratio of the vertical distance to the horizontal distance is called slope

$$\text{Slope} = \frac{\text{Vertical Hight}}{\text{Horizontal distance}}$$

$$\text{Slope} = \frac{AB}{BC}$$

$$\text{Slope} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$



$$\therefore m = \tan\theta$$

[m= Slope]

$$\text{Gradient:} = \frac{\text{The increase in}-y}{\text{The increase in}-x}$$

$$= \frac{\text{Verticale hight}}{\text{Horizontal distance}}$$

$$= \text{Slope}$$

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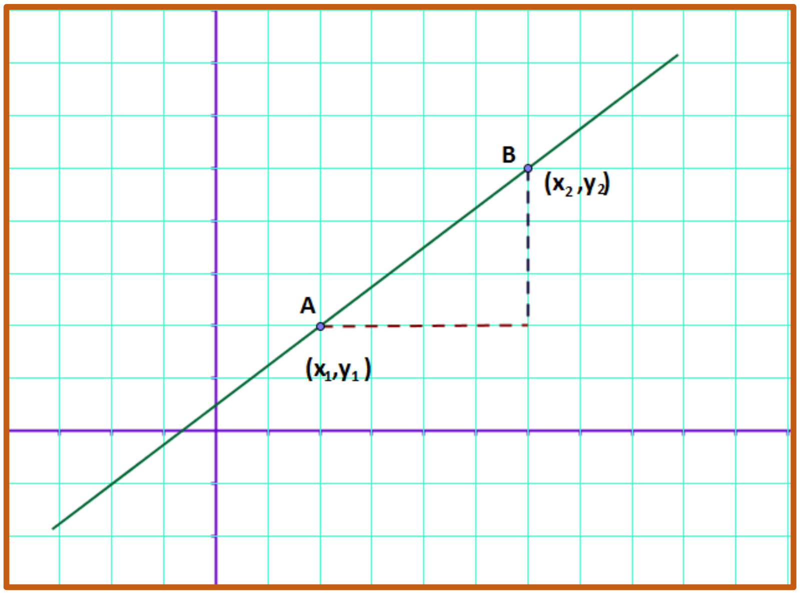
Value of angle of inclination	Value of slope
$\theta = 0^0$	0
$0^0 < \theta < 90^0$	Positive number
$\theta = 90^0$	Not defined
$90^0 < \theta < 180^0$	Negative number

- **Slope of a straight line passing through two given points:**



$$\text{Slope} = \frac{\text{Difference of ordinates of given points}}{\text{Difference of their abscissa}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- If the slopes of two lines are m_1 and m_2 ,

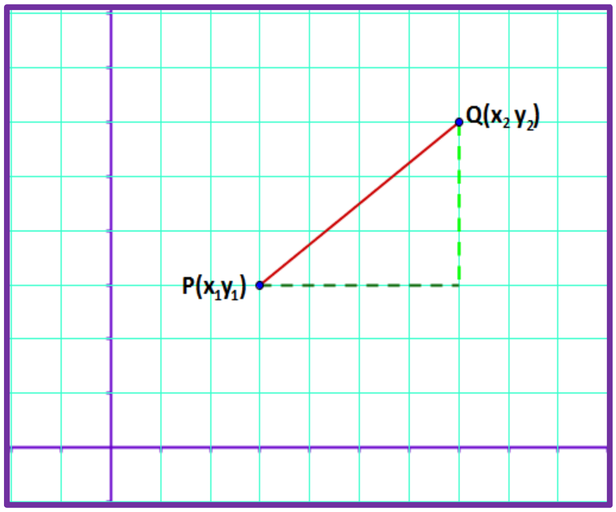
$m_1 = m_2$ ⇒ **The lines are parallel**

$m_1 \text{x } m_2 = -1$ ⇒ **The lines are perpendicular**

- **The equation of a line with slope 'm' and whose y-intercept is 'c'**

$$y = mx + c$$

Distance between two points



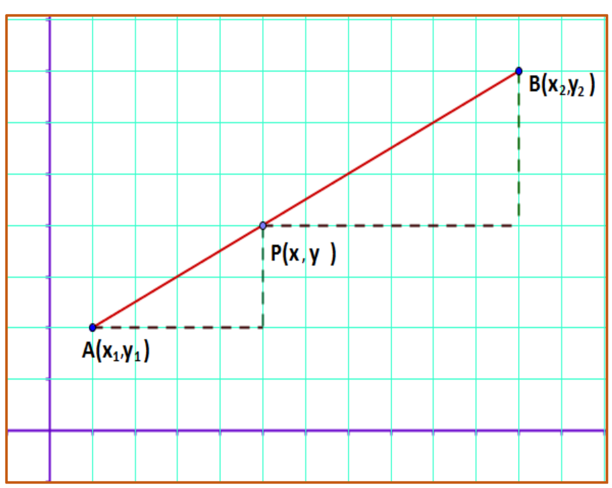
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point in a plane from the Origin:

$$d = \sqrt{x^2 + y^2}$$

Section formula:

P divides the line segment AB in the ratio m:n P(x,y)



$$P(x,y) = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$$

Mid Point formula

If P' is the midpoint of the line segment AB,

$$P(x,y) = \left[\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right]$$

EXERCISE 14.1

1. Find the slope of the line whose inclination is.
   (i). 90^0 (ii). 45^0 (iii). 30^0 (iv). 0^0
   (i). 90^0
   $\tan 90^0$ = ND
   (ii). 45^0
   $\tan 45^0 = 1$
   (iii). 30^0
   $\tan 30^0 = \frac{1}{\sqrt{3}}$
   (iv). 0^0
   $\tan 0^0 = 0$

2. Find the angles of inclination of straight lines whose slopes are.
   (i). $\sqrt{3}$ (ii). 1 (iii). $\frac{1}{\sqrt{3}}$
   (i). $\sqrt{3}$
   $\sqrt{3} = \tan 60^0$
   $\Rightarrow \theta = 60^0$
   (ii). 1
   $1 = \tan 45^0$
   $\Rightarrow \theta = 45^0$
   (iii). $\frac{1}{\sqrt{3}}$
   $1 = \tan 30^0$
   $\Rightarrow \theta = 30^0$

3. Find the slope of the line joining the points.
   (i). (-4,1) and (-5,2) (ii). (4,-8) and (5,-2) (iii). (0,0) and ($\sqrt{3}$,3)
   (iv). (-5,0) and (0,-7) (v). (2a,3b) and (a,-b)
   (i). (-4,1) and (-5,2)
   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
   $$m = \frac{2 - 1}{-5 - (-4)}$$
   $$m = \frac{1}{-5 + 4}$$
   $$\mathbf{m = -1}$$

   (ii). (4,-8) and(5,-2)
   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
   $$m = \frac{-2 - (-8)}{5 - 4}$$
   $$m = \frac{-2 + 8}{1}$$
   $$\mathbf{m = 6}$$

   (iii). (0,0) and ($\sqrt{3}$,3)
   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
   $$m = \frac{3 - 0}{\sqrt{3} - 0}$$
   $$\mathbf{m = \frac{3}{\sqrt{3}}}$$

   (iv). (-5,0) and (0,-7)
   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
   $$m = \frac{-7 - 0}{0 - (-5)}$$
   $$\mathbf{m = \frac{-7}{5}}$$

   (v). (2a,3b) and (a,-b)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-b - 3b}{a - 2a)}$$
$$m = \frac{-4b}{-a}$$
$$\mathbf{m} = \frac{\mathbf{4b}}{\mathbf{a}}$$

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4. Find whether the lines drawn through the two pairs of points are parallel or perpendicular.
(i). (5,2), (0,5) and (0,0),(-5,3) (ii). (3,3), (4,6) and (4,1),(6,7)
iii). (4,7), (3,5) and (-1,7),(1,6) (iv). (-1,-2), (1,6) and (-1,1),(-2,-3)

(i). (5,2), (0,5) and (0,0),(-5,3)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_1 = \frac{5 - 2}{0 - 5}$$
$$m_1 = \frac{3}{-5}$$
$$m_2 = \frac{3 - 0}{-5 - 0}$$
$$m_2 = \frac{3}{-5}$$
$$\mathbf{m_1 = m_2}$$

∴ Lines are parallel

(ii). (3,3), (4,6) and (4,1),(6,7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_1 = \frac{6 - 3}{4 - 3}$$
$$m_1 = \frac{3}{1}$$
$$m_1 = 3$$
$$m_2 = \frac{7 - 1}{6 - 4}$$
$$m_2 = \frac{6}{2}$$
$$m_1 = 3$$

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$\mathbf{m_1 = m_2}$

∴ Lines are parallel

iii). (4,7), (3,5) and (-1,7),(1,6)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{5-7}{3-4}$

$m_1 = \frac{-2}{-1}$

$m_1 = 2$

$m_2 = \frac{6-7}{1-(-1)}$

$m_2 = \frac{-1}{1+1}$

$m_2 = \frac{-1}{2}$

$m_1 x\, m_2$

$2x\frac{-1}{2}$ = -1

∴ Lines are perpendicular

iv). (-1,-2), (1,6) and (-1,1),(-2,-3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{6-(-2)}{1-(-1)}$

$m_1 = \frac{8}{2}$

$m_1 = 4$

$m_2 = \frac{-3-1}{-2-(-1)}$

$m_2 = \frac{-4}{-1}$

$m_2 = 4$

$m_1 = m_2$

∴ Lines are parallel

5. Find the slope of the line perpendicular to the line joining the points.
(i). (1,7) and (-4,3) (ii). (2,-3) and (1,4)

(i). (1,7) and (-4,3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{3-7}{-4-1}$

$m_1 = \frac{-4}{-5}$

$m_1 = \frac{4}{5}$

If the lines are perpendicular, then $m_1 x\, m_2 = -1$

$\frac{4}{5} x\, m_2 = -1$

$\Rightarrow m_2 = -1 x \frac{5}{4}$

$\Rightarrow m_2 = \frac{-5}{4}$

(ii). (2,-3) and (1,4)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{4-(-3)}{1-2}$

$m_1 = \frac{4+3}{-1}$

$m_1 = -7$

If lines are perpendicular,then $m_1 x\, m_2 = -1$

$-7 x\, m_2 = -1$

$\Rightarrow m_2 = \frac{-1}{-7}$

$\Rightarrow m_2 = \frac{1}{7}$

6. Find the slope of the line parallel to the line joining the
(i). (-4,3) and (2,5) (ii). (1,-5) and (7,1)

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(i). (-4,3) and (2,5)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{5-3}{2-(-4)}$$

$$m_1 = \frac{2}{6}$$

$$m_1 = \frac{1}{3}$$

If lines are parallel,then $m_1 = m_2$

$\Rightarrow m_2 = \frac{1}{3}$

(ii). (1,-5) and (7,1)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{1-(-5)}{7-1}$$

$$m_1 = \frac{6}{6}$$

$$m_1 = 1$$

If lines are parallel,then $m_1 = m_2$

$\Rightarrow m_2 = 1$

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7. A line passing through the points (2, 7) and (3, 6) is parallel to a line joining (9, a) and (11, 3). Find 'a'.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{6-7}{3-2}$$

$$m_1 = \frac{-1}{1}$$

$$m_1 = -1$$

$$m_2 = \frac{3-a}{11-9}$$

$$m_2 = \frac{3-a}{2}$$

If lines are parallel,then $m_1 = m_2$

$\Rightarrow -1 = \frac{3-a}{2}$

$\Rightarrow -2 = 3 - a$

$\Rightarrow a = 3 + 2$
$\Rightarrow a = 5$

8. A line passing through the points (1, 0) and (4, 3) is perpendicular to the line joining (–2, –1) and (m, 0). Find the value of 'm'

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{3 - 0}{4 - 1}$

$m_1 = \frac{3}{3}$

$m_1 = 1$

$m_2 = \frac{0 - (-1)}{m - (-2)}$

$m_2 = \frac{1}{m + 2}$

If lines are perpendicular,then $m_1 \times m_2 = -1$

$\Rightarrow 1 \times \frac{1}{m+2} = -1$
$\Rightarrow 1 = -(m + 2)$
$\Rightarrow 1 = -m - 2$
$\Rightarrow m = -2 - 1$
$\Rightarrow m = -3$

EXERCISE – 14.2

1. Find the equation of the line whose angle of inclination and y-intercept are given.

(i) $\theta = 60°$, y-intercept is – 2 (ii) $\theta = 45°$, y-intercept is 3

(i) $\theta = 60°$, y-intercept is – 2

$\theta = 60^0$

$\Rightarrow m = \tan 60^0 = \sqrt{3}$

y-intercept is – 2

$y = mx + c$

$\therefore y = \sqrt{3}x + (-2)$

$\Rightarrow y = \sqrt{3}x - 2$

(ii). $\theta = 45°$, y-intercept is 3

$\theta = 45^0$

$\Rightarrow$ m = $\tan 45^0 = 1$

y-intercept is 3

y = mx + c

$\therefore$ y = x + 3

2. Find the equation of the line whose slope and y-intercept are given.

(i) Slope = 2, y-intercept = – 4

(ii). Slope = $\frac{-2}{3}$, y-intercept = -$\frac{1}{2}$

(ii) Slope = –2, y-intercept = 3

(i) Slope = 2, y-intercept = – 4

y = mx + c

$\Rightarrow$ y = 2x - 4

(ii). Slope = $\frac{-2}{3}$, Y- y-intercept = -$\frac{1}{2}$

y = mx + c

$\Rightarrow$ y = $\left(\frac{-2}{3}\right)x - \left(-\frac{1}{2}\right)$

$\Rightarrow$ y = $\frac{-2}{3}x + \frac{1}{2}$

$\Rightarrow$ y = $\frac{-4x+3}{6}$

$\Rightarrow$ 6y = $-4x + 3$

(iii) Slope = –2, y-intercept = 3

y = mx + c

$\Rightarrow$ y = $(-2)x + 3$

$\Rightarrow$ y = $-2x + 3$

3. Find the slope and y-intercept of the lines

(i) 2x + 3y = 4 (ii) 3x = y

(iii) x – y + 5 = 0 (iv) 3x – 4y = 5

(i). 2x + 3y = 4

3y = -2x + 4

y = $\frac{-2}{3}x + \frac{4}{3}$

slope (m) = $\frac{-2}{3}$; y-intercept (c) = $\frac{4}{3}$

(ii). 3x = y

3y = -2x + 4

y = $\frac{-2}{3}x + \frac{4}{3}$

slope (m) = 3; y-intercept (c) = 0

(iii). x - y + 5 = 0

-y = -x – 5

y = x + 5

slope (m) = 1; y-intercept (c) = 5

(iv). 3x - 4y = 5

-4y = -3x + 5

y = $\frac{-3}{-4}x + \frac{5}{-4}$

y = $\frac{3}{4}x + \left(\frac{-5}{4}\right)$

slope (m) = $\frac{3}{4}$; y-intercept (c) = $\frac{-5}{4}$

3. Is the line x = 2y parallel to 2x – 4y + 7 = 0.

[Hint : Parallel lines have same slopes]

x = 2y

$\Rightarrow$y = $\frac{x}{2}$

$m_1 = \frac{1}{2}$

2x – 4y + 7 = 0

$\Rightarrow$-4y = -2x + 7

$\Rightarrow$y = $\frac{-2}{-4}$x + $\frac{7}{-4}$

$\Rightarrow$y = $\frac{1}{2}x + \left(-\frac{7}{4}\right)$

$m_2 = \frac{1}{2}$

$\Rightarrow m_1 = m_2$

∴ Lines are parallel

1. Show that the line 3x + 4y + 7 = 0 and 28x – 21y + 50 = 0 are perpendicular to each other.
[Hint : For perpendicular lines, $m_1 x m_2 = -1$]
3x + 4y + 7 = 0
4y = -3x – 7
$y = \frac{-3}{4}x + \frac{-7}{4}$

$m_1 = \frac{-3}{4}$
28x – 21y + 50 = 0
– 21y = -28x - 50
$y = \frac{-28}{-21}x + \left(\frac{-50}{-21}\right)$
$y = \frac{4}{3}x + \left(\frac{50}{21}\right)$

$m_2 = \frac{4}{3}$
$m_1 \, x \, m_2$
$= \frac{-3}{4} \, x \, \frac{4}{3}$
$= -1$

∴ Lines are perpendicular

EXERCISE 14.3

1. Find the distance between the following pairs of points
(i) (8, 3) and (8, –7) (ii) (1, –3) and (–4, 7)
(iii) (–4, 5) and (–12,3) (iv) (6, 5) and (4, 4)
(v) (2, 0) and (0, 3) (vi) (2, 8) and (6, 8)

(i). (8, 3) and (8, -7)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(8-8)^2 + (-7-3)^2}$$

$$d = \sqrt{0^2 + (-10)^2}$$

$$d = \sqrt{100}$$

$$\boldsymbol{d = 10 units}$$

(ii). (1, -3) and (-4, 7)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-4 - 1)^2 + [7 - (-3)]^2}$

$d = \sqrt{(-5)^2 + (10)^2}$

$d = \sqrt{25 + 100}$

$d = \sqrt{125}$

$\boldsymbol{d = 5\sqrt{5}\ units}$

(iii). (-4, 5) and (-12, 3)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{[-12 - (-4)]^2 + (3 - 5)^2}$

$d = \sqrt{(-8)^2 + (-2)^2}$

$d = \sqrt{64 + 4}$

$d = \sqrt{68}$

$\boldsymbol{d = 2\sqrt{17}\ units}$

(iv). (6, 5) and (4, 4)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4 - 6)^2 + (4 - 5)^2}$

$d = \sqrt{(-2)^2 + (-1)^2}$

$d = \sqrt{4 + 1}$

$\boldsymbol{d = \sqrt{5}\ units}$

(v). (2, 0) and (0, 3)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(0 - 2)^2 + (3 - 0)^2}$

$d = \sqrt{(-2)^2 + (3)^2}$

$d = \sqrt{4+9}$

$\boldsymbol{d = \sqrt{13}\ units}$

(vi). (2, 8) and (6, 8)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(6-2)^2 + (8-8)^2}$

$d = \sqrt{(4)^2 + (0)^2}$

$d = \sqrt{16}$

$\boldsymbol{d = 4\ units}$

(vii). (a, b) and (c, b)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(c-a)^2 + (b-b)^2}$

$d = \sqrt{(c-a)^2 + (0)^2}$

$d = \sqrt{(c-a)^2}$

$\boldsymbol{d = (c-a)\ units}$

(viii). ($\cos\theta$, -$\sin\theta$) and ($\sin\theta$, $-\cos\theta$)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(\sin\theta - \cos\theta)^2 + [-\cos\theta - (-\sin\theta)]^2}$

$d = \sqrt{(\sin\theta - \cos\theta)^2 + (-\cos\theta + \sin\theta)^2}$

$d = \sqrt{(\sin\theta - \cos\theta)^2 + (\sin\theta - \cos\theta)^2}$

$d = \sqrt{2(\sin\theta - \cos\theta)^2}$

$\boldsymbol{d = \sqrt{2}\ (\sin\theta - \cos\theta)\ units}$

2. Find the distance between the origin and the point
   (i) (–6, 8) (ii) (5, 12) (iii) (–8, 15)

(i). (-6, 8)

$\mathrm{d} = \sqrt{x^2 + y^2}$

$d = \sqrt{(-6)^2 + 8^2}$

$d = \sqrt{36 + 64}$

$d = \sqrt{100}$

$\mathbf{d = 10}\boldsymbol{units}$

(ii). (5, 12)

$d = \sqrt{x^2 + y^2}$

$d = \sqrt{5^2 + 12^2}$

$d = \sqrt{25 + 144}$

$d = \sqrt{169}$

$\mathbf{d = 13}\boldsymbol{units}$

(ii). (-8, 15)

$d = \sqrt{x^2 + y^2}$

$d = \sqrt{(-8)^2 + 15^2}$

$d = \sqrt{64 + 225}$

$d = \sqrt{289}$

$\mathbf{d = 17}\boldsymbol{units}$

3. (i) The distance between the points (3, 1) and (0, x) is 5 units. Find x.
   (ii) A point P(2, –1) is equidistant from the points (a, 7) and (– 3, a). Find 'a'.
   (iii) Find a point on y-axis which is equidistant from the points (5, 2) and (–4, 3)

(i) The distance between the points (3, 1) and (0, x) is 5 units. Find x.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(0 - 3)^2 + (x - 1)^2}$

$5 = \sqrt{(-3)^2 + (x - 1)^2}$

$25 = 9 + (x-1)^2$

$(x-1)^2 = 25 - 9$

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$x = 1 \pm 4$

$x = 1 + 4$ or $x = 1 - 4$

$\boldsymbol{x = 5}$ **or** $\boldsymbol{x = -3}$

(ii) A point P(2, –1) is equidistant from the points (a, 7) and (– 3, a). Find 'a'.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_1 = \sqrt{(a-2)^2 + [7-(-1)]^2}$

$d_1 = \sqrt{(a-2)^2 + 8^2}$

$d_1 = \sqrt{a^2 - 4a + 4 + 64}$

$d_1 = \sqrt{a^2 - 4a + 68}$

$\boldsymbol{d_1^2 = a^2 - 4a + 68}$

$d_2 = \sqrt{(-3-2)^2 + [a-(-1)]^2}$

$d_2 = \sqrt{(-5)^2 + (a+1)^2}$

$d_2 = \sqrt{25 + a^2 + 2a + 1}$

$d_2 = \sqrt{a^2 + 2a + 26}$

$\boldsymbol{d_2^2 = a^2 + 2a + 26}$

$d_1^2 = d_2^2$

$a^2 - 4a + 68 = a^2 + 2a + 26$

$-4a - 2a = 26 - 68$

$-6a = -42$

$\Rightarrow a = \frac{-42}{-6}$

$\Rightarrow \boldsymbol{a} = \mathbf{7}$

(iii) Find a point on y-axis which is equidistant from the points (5, 2) and (–4, 3)

The point on y-axis P(0, y)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_1 = \sqrt{(5-0)^2 + (2-y)^2}$

$d_1 = \sqrt{25 + 4 + y^2 - 4y}$

$d_1 = \sqrt{29 + y^2 - 4y}$

$d_1^2 = 29 + y^2 - 4y$

$d_2 = \sqrt{(-4-0)^2 + (3-y)^2}$

$d_2 = \sqrt{16 + 9 - 6y + y^2}$

$d_2 = \sqrt{25 - 6y + y^2}$

$d_2^2 = 25 - 6y + y^2$

$d_1^2 = d_2^2$

$29 + y^2 - 4y = 25 - 6y + y^2$

$29 - 4y = 25 - 6y$

$6y - 4y = 25 - 29$

$2y = -4$

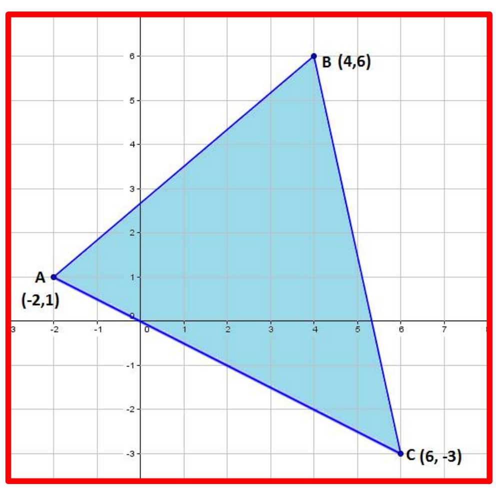
$y = -2$

∴The required point is P(0,-2)

4. Find the perimeter of the triangles whose vertices have the following coordinates

(i) (–2, 1), (4, 6), (6, –3) (ii) (3, 10), (5, 2), (14, 12)

(i). (-2, 1), (4, 6), (6, -3)



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{[4-(-2)]^2 + (6-1)^2}$

$AB = \sqrt{6^2 + 5^2}$

$AB = \sqrt{36 + 25}$

$AB = \sqrt{61}\ units$

$BC = \sqrt{(6-4)^2 + (-3-6)^2}$

$BC = \sqrt{2^2 + (-9)^2}$

$BC = \sqrt{4 + 81}$

$BC = \sqrt{85} units$

$AC = \sqrt{[6 - (-2)]^2 + (-3 - 1)^2}$

$AC = \sqrt{8^2 + (-4)^2}$

$AC = \sqrt{64 + 16}$

$AC = \sqrt{80}\ units$

$\therefore$ The perimeter = AB + BC + AC $= (\sqrt{61} + \sqrt{85} + \sqrt{80})$ units

(ii). (3, 10), (5, 2), (14, 12)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(5 - 3)^2 + (2 - 10)^2}$

$AB = \sqrt{2^2 + (-8)^2}$

$AB = \sqrt{4 + 64}$

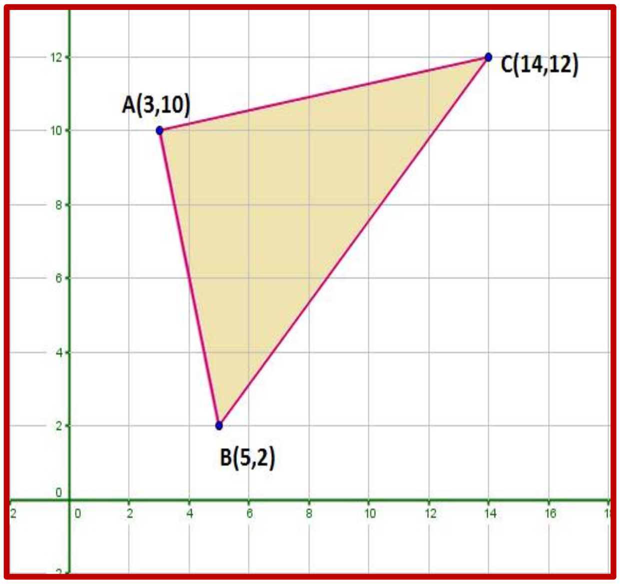
$AB = \sqrt{68}\ units$

$BC = \sqrt{(14 - 5)^2 + (12 - 2)^2}$

$BC = \sqrt{9^2 + 10^2}$

$BC = \sqrt{81 + 100}$

$BC = \sqrt{181} units$



$AC = \sqrt{(14 - 3)^2 + (12 - 10)^2}$

$AC = \sqrt{11^2 + 2^2}$

$AC = \sqrt{121 + 4}$

$AC = \sqrt{125}\ units$

$\therefore$ The perimeter = AB + BC + AC $= (\sqrt{68} + \sqrt{181} + \sqrt{125})$ units

5. Prove that the points A(1, –3), B(–3, 0) and C(4, 1) are the vertices of a right isosceles triangle..

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{[4 - (-3)]^2 + (1 - 0)^2}$

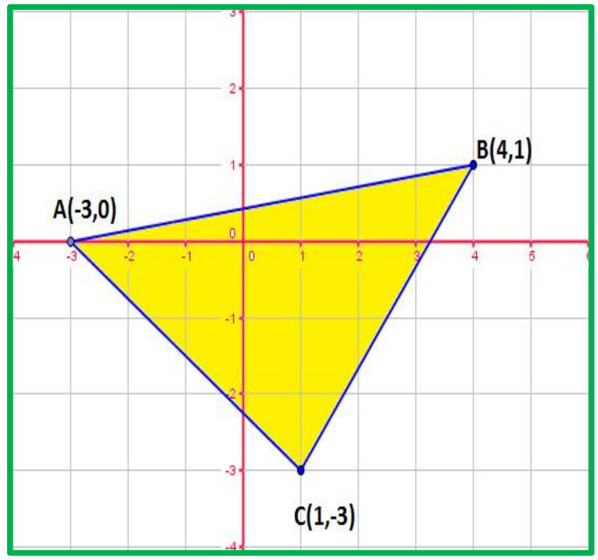
$AB = \sqrt{7^2 + 1^2}$

$AB = \sqrt{49 + 1}$

$AB = \sqrt{50}\ units$

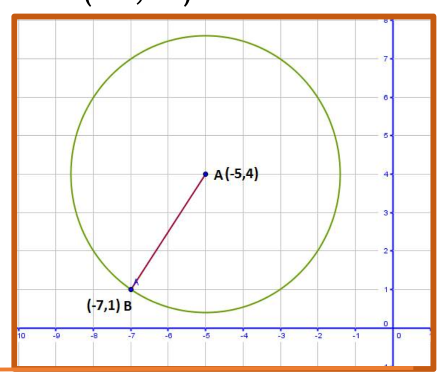
$BC = \sqrt{(1 - 4)^2 + (-3 - 1)^2}$

$BC = \sqrt{(-3)^2 + (-4)^2}$
$BC = \sqrt{9 + 16}$
$BC = \sqrt{25} units$
$BC = 5 units$
$AC = \sqrt{[1-(-3)]^2 + (-3-0)^2}$
$AC = \sqrt{4^2 + (-3)^2}$
$AC = \sqrt{16 + 9}$
$AC = \sqrt{25}\ units$
$AC = 5\ units$



$AB^2 = (\sqrt{50})^2$
$AB^2 = 50$ ----------(1)
$BC^2 = 5^2$
$BC^2 = 25$-----------(2)
$AC^2 = 5^2$
$AC^2 = 25$-----------(3)
From (1), (2) and (3)
$AB^2 = BC^2 + AC^2$
∴ ΔABC is a right angled triangle [∵Inverse of Pythagoras theorem]
Side BC = side AC = 5
∴ ΔABC is a right isosceles triangle
A, B, C are vertices of righta right isosceles triangle

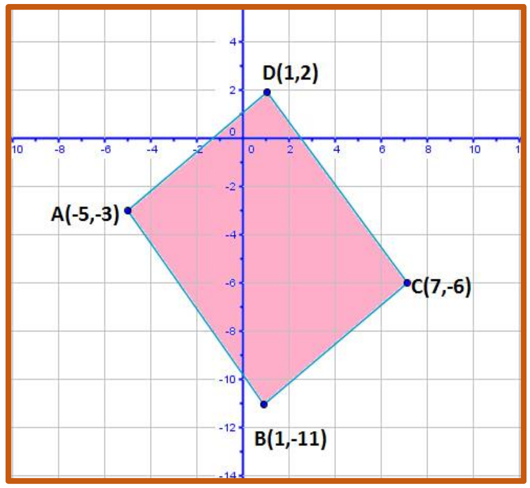
6. Find the radius of a circle whose centre is (–5, 4) and which passes through the point (–7, 1).



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{[-7-(-5)]^2 + (1-4)^2}$
$AB = \sqrt{(-7+5)^2 + (-3)^2}$
$AB = \sqrt{(-2)^2 + (-3)^2}$
$AB = \sqrt{4 + 9}$
Radius AB = $\sqrt{13}$ units

7. Prove that each of the set of coordinates are the vertices of parallelograms.

(i) (–5, –3), (1, –11), (7, –6), (1, 2) (ii) (4, 0), (–2, –3), (3, 2), (–3, –1) (i). (-5,-3),(1,-11),(7,-6),(1,2)



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{[1-(-5)]^2 + [-11-(-3)]^2}$

$AB = \sqrt{6^2 + (-8)^2}$

$AB = \sqrt{36 + 64}$

$AB = \sqrt{100}$

AB = 10 units

$BC = \sqrt{(7-1)^2 + [-6-(-11)]^2}$

$BC = \sqrt{6^2 + 5^2}$

$BC = \sqrt{36 + 25}$

$BC = \sqrt{61}$ units

$CD = \sqrt{(1-7)^2 + [2-(-6)]^2}$

$CD = \sqrt{(-6)^2 + 8^2}$

$CD = \sqrt{36 + 64}$

$CD = \sqrt{100}$

CD = 10 units

$AD = \sqrt{[1-(-5)]^2 + [2-(-3)]^2}$

$AB = \sqrt{6^2 + 5^2}$

$AB = \sqrt{36 + 25}$

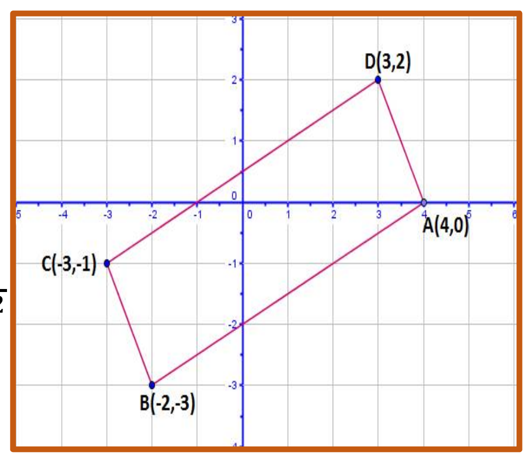
AD= $\sqrt{61}$ units

⇒ AB=CD and BC = AD

∴ A, B, C and D are the vertices of parallelogram

[∵ Opposite sides are equal]

(ii). (4, 0),(-2,-3),(3,2),(-3,-1)



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(-2-4)^2 + (-3-0)^2}$

$AB = \sqrt{(-6)^2 + (-3)^2}$

$AB = \sqrt{36 + 9}$

$AB = \sqrt{45}$ units

$BC = \sqrt{[(-3-(-2)]^2 + [-1-(-3)]^2}$

$BC = \sqrt{(-1)^2 + 2^2}$

$BC = \sqrt{1 + 4}$

$BC = \sqrt{5}$ units

$CD = \sqrt{[3-(-3)]^2 + [2-(-1)]^2}$

$CD = \sqrt{6^2 + 3^2}$

$CD = \sqrt{36+9}$

$CD = \sqrt{45}$ units

$AD = \sqrt{(3-4)^2 + (2-0)^2}$

$AB = \sqrt{1^2 + 2^2}$

$AB = \sqrt{1+4}$

$AD = \sqrt{5}$ units

⇒ AB=CD and BC = AD

∴ A, B, C and D are vertices of a parallelogram

[∵ Opposite sides are equal]

8. The coordinate of vertices of triangles are given. Identify the types of triangles.

(i) (2, 1) (10, 1) (6, 9) (ii) (1, 6) (3, 2) (10, 8)

(iii) (3, 5) (–1, 1) (6, 2) (iv) (3, –3) (3, 5) (11, –3)

(i). (2,1), (10,1),(6,9)

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AB = \sqrt{(2-6)^2 + (1-9)^2}$

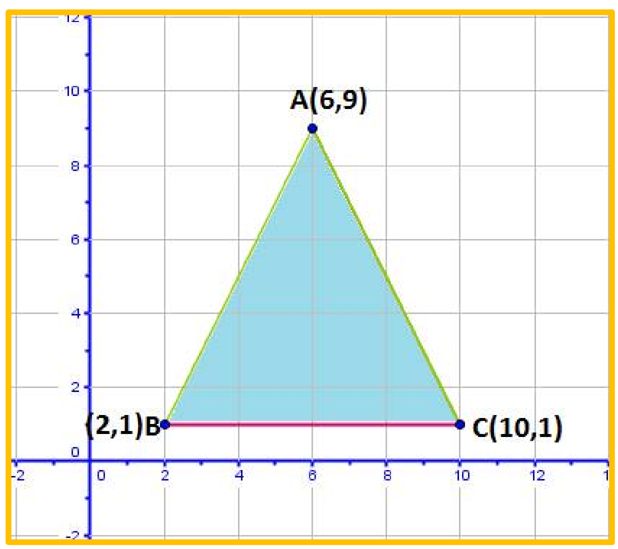
$AB = \sqrt{(-4)^2 + (-8)^2}$

$AB = \sqrt{16+64}$

$AB = \sqrt{80}\ units$

$AB = \sqrt{16x5}\ units$

$AB = 4\sqrt{5}\ units$



$BC = \sqrt{(10-2)^2 + (1-1)^2}$

$BC = \sqrt{8^2 + 0^2}$

$BC = \sqrt{64+0}$

$BC = \sqrt{64}$

$BC = 8$

$BC = 8units$

$AC = \sqrt{(10-6)^2 + (1-9)^2}$

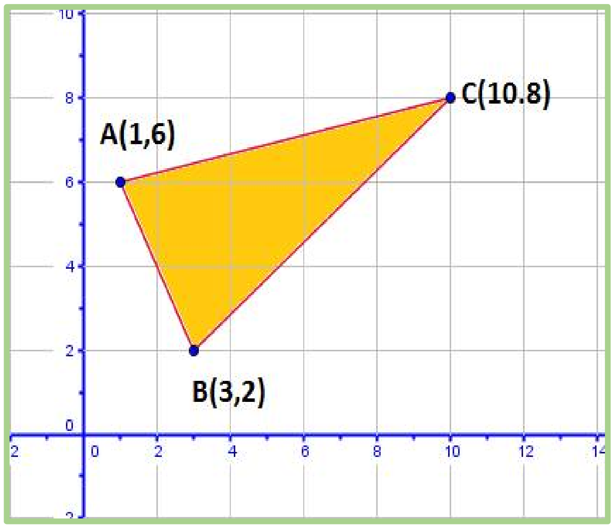
$AC = \sqrt{4^2 + (-8)^2}$

$AC = \sqrt{16+64}$

$AC = \sqrt{80}$

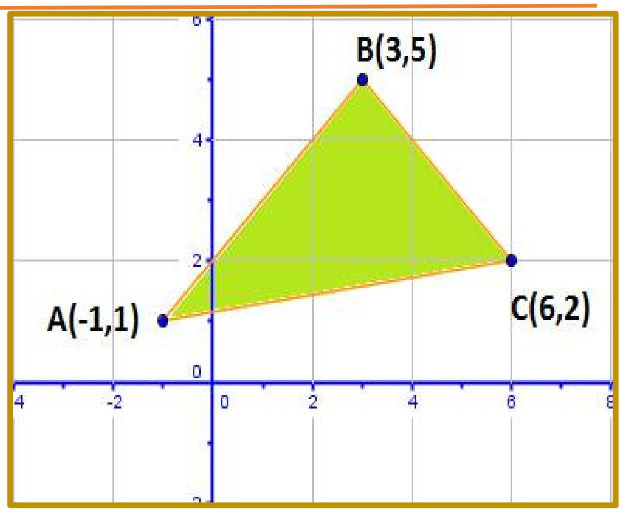
$AC = \sqrt{16x5}$
$AC = 4\sqrt{5}\ units$
$AB = AC$
$\therefore \Delta ABC$ is isosceles

(ii). (1,6), (3,2),(10,8)



$d \ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{(3-1)^2 + (2-6)^2}$
$AB = \sqrt{2^2 + (-4)^2}$
$AB = \sqrt{4 + 16}$
$AB = \sqrt{20}$
$AB = \sqrt{4x5}\ units$
$AB = 2\sqrt{5}\ units$
$BC = \sqrt{(10-3)^2 + (8-2)^2}$
$BC = \sqrt{7^2 + 6^2}$
$BC = \sqrt{49 + 36}$
$BC = \sqrt{85}\ units$
$AC = \sqrt{(10-1)^2 + (8-6)^2}$
$AC = \sqrt{9^2 + 2^2}$
$AC = \sqrt{81 + 4}$
$AC = \sqrt{85}\ units$
BC = AC
$\therefore \Delta ABC$ is isosceles

(iii). (3,5), (-1,1),(6,2)

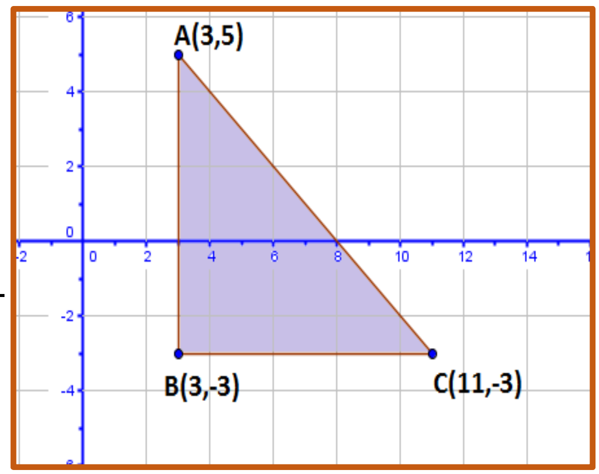


$d \ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{[3-(-1)]^2 + (5-1)^2}$
$AB = \sqrt{4^2 + 4^2}$
$AB = \sqrt{16 + 16}$
$AB = \sqrt{32}$
$AB^2 = 32$
$BC = \sqrt{(6-3)^2 + (2-5)^2}$
$BC = \sqrt{3^2 + (-3)^2}$
$BC = \sqrt{9 + 9}$
$BC = \sqrt{18}$
$BC^2 = 18$

$AC = \sqrt{[6-(-1)]^2 + (2-1)^2}$
$AC = \sqrt{7^2 + 1^2}$
$AC = \sqrt{49 + 1}$
$AC = \sqrt{50}$
$AC^2 = 50$
$AC^2 = AB^2 + BC^2$
∴ ΔABC is right angled[∵Inverse of Pythagoras theorem]

(iv). (3,-3), (3, 5),(11,-3)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{(3-3)^2 + (-3-5)^2}$
$AB = \sqrt{0^2 + (-8)^2}$
$AB = \sqrt{0 + 64}$
$AB = \sqrt{64}$
$AB^2 = 64$



$BC = \sqrt{(11-3)^2 + [-3-(-3)]^2}$
$BC = \sqrt{8^2 + 0^2}$
$BC = \sqrt{64 + 0}$
$BC = \sqrt{64}$
$BC^2 = 64$
$AC = \sqrt{(11-3)^2 + (-3-5)^2}$
$AC = \sqrt{8^2 + (-8)^2}$
$AC = \sqrt{64 + 64}$
$AC = \sqrt{128}$
$AC^2 = 128$
$AC^2 = AB^2 + BC^2$
∴ ΔABC is right angles isosceles[∵inverse of Pythagoras theorem]

EXERCISE 14.4

1. In what ratio does the point (-2, 3) divide the line segment joining the points (-3, 5) and (4, -9) ?

$P(x,y) = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$

$P(-2,3) = \left[\frac{4m-3n}{m+n}, \frac{-9m+9n}{m+n}\right]$

$\frac{4m-3n}{m+n} = -2$

$4m - 3n = -2(m + n)$

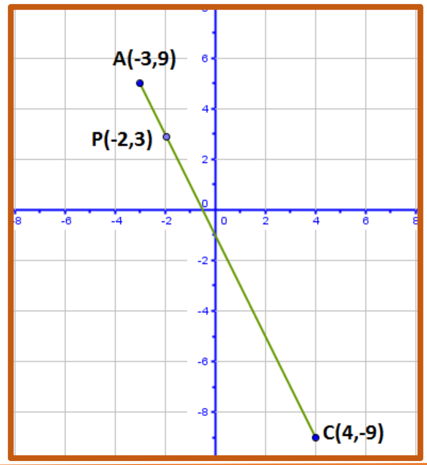
$4m - 3n = -2m - 2n$

$4m + 2m = 3n - 2n$

$6m = n$

$\frac{m}{n} = \frac{1}{6}$

$\therefore$ **m:n = 1: 6**



2. In the point C (1,1) divides the line segment joining A(-2,7) and B in the ratio 3:2, find the coordinates of B

$P(x,y) = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$

$C(1,1) = \left[\frac{3.x_2+2(-2)}{3+2}, \frac{3y_2+2(7)}{3+2}\right]$

$C(1,1) = \left[\frac{3.x_2-4}{5}, \frac{3y_2+14}{5}\right]$

$\Rightarrow 3x_2 - 4 = 5$

$\Rightarrow 3x_2 = 9$

$\Rightarrow x_2 = 3$

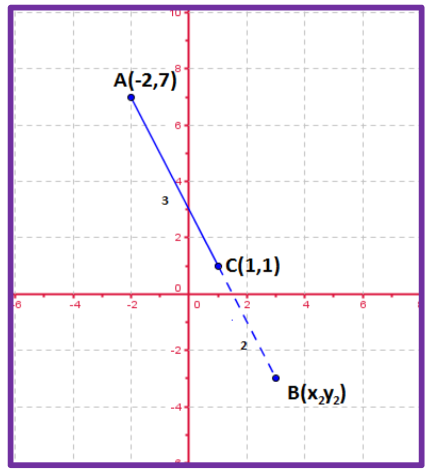
$\frac{3y_2+14}{5} = 1$

$\Rightarrow 3y_2 + 14 = 5$

$\Rightarrow 3y_2 = -9$

$\Rightarrow y_2 = -3$

$\therefore$ **B (x_2, y_2) = (3,-3)**



3. Find the ratio in which the point (-1, k) divides the line joining the points (-3, 10) and (6, -8), and find the value of k.

$P(x,y) = \left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$

$P(-1,k) = \left[\frac{6m-3n}{m+n}, \frac{-8m+10n}{m+n}\right]$

$\Rightarrow \frac{6m-3n}{m+n} = -1$

$\Rightarrow 6m - 3n = -1(m + n)$

$\Rightarrow 6m - 3n = -m - n)$

$\Rightarrow 6m + m = 3n - n)$

$\Rightarrow 7m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{7}$

$\Rightarrow \boldsymbol{m : n = 2 : 7}$

$\frac{-8m+10n}{m+n} = \text{k}$

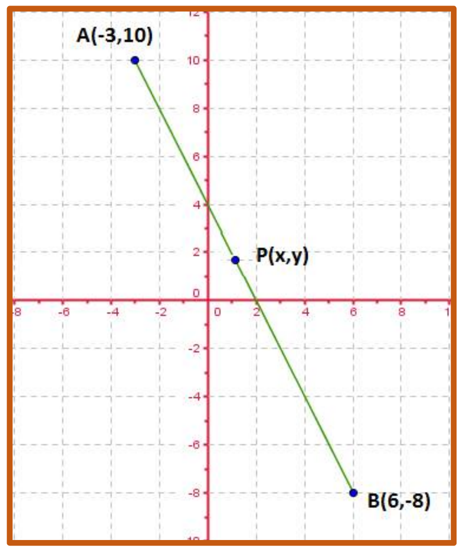
$-8\text{x}2 + 10\text{x}7 = \text{k}(2 + 7)$

$-16 + 70 = 9k$

$9k = 54$

$\mathbf{k = 6}$

4. Find the coordinaes of the midpoint of the line joining the points (-3, 10) and (6, -8).



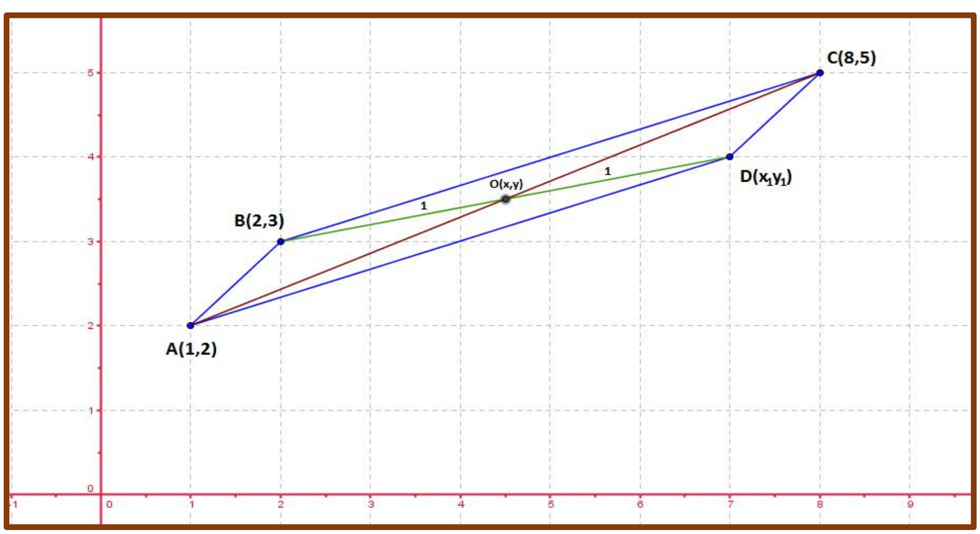
$P(x,y) = \left[\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right]$

$P(x,y) = \left[\frac{6-3}{2}, \frac{-8+10}{2}\right]$

$P(x,y) = \left[\frac{3}{2}, \frac{2}{2}\right]$

$P(x,y) = \left(\frac{3}{2}, 1\right)$

4. Three consecutive vertices of a parallelogram are A(1,2), B(2,3) and C(8,5). Find the fourth vertex. (Hint : diagonals of a parallelogram bisect each other]



$P(x,y) = \left[\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right]$

$O(x,y) = \left[\frac{8+1}{2}, \frac{5+2}{2}\right]$

$O(x,y) = \left(\frac{9}{2}, \frac{7}{2}\right)$

$O\left(\frac{9}{2}, \frac{7}{2}\right) = \left[\frac{x_1+2}{2}, \frac{y_1+3}{2}\right]$

$\Rightarrow \frac{x_1+2}{2} = \frac{9}{2}$

$\Rightarrow x_1 = \frac{9}{2} x 2 - 2$

$\Rightarrow \boldsymbol{x_1 = 7}$

$\Rightarrow \frac{y_1+3}{2} = \frac{7}{2}$

$\Rightarrow y_1 = \frac{7}{2} x 2 - 3$

$\Rightarrow \boldsymbol{y_1 = 4}$

∴ **$D(x_1,y_1) = (7,4)$**